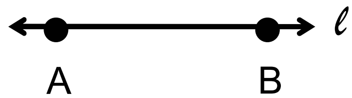
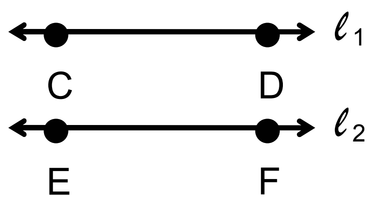
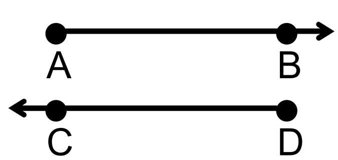
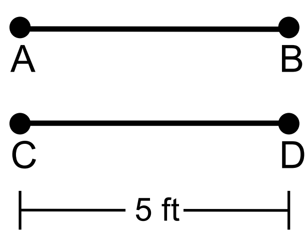
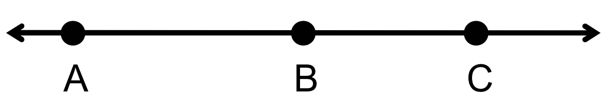
Sect 9.1 - Some Basic Definitions

Objective a: Understanding points and lines:

Definition	**Illustration**	**Notation**
A point is a location in space. It is indicated by making a dot. Points are typically labeled with capital letters next to the dot.		Point A and point B
A line is determined by two different points and extends infinitely in **two** directions. A line can be labeled by small case letter or by two different points. If a picture contains two or more lines, subscripts can be used to denote the lines.	 	Line AB or line ℓ or $\overleftrightarrow{AB}$ $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ or lines ℓ_1 and ℓ_2
A ray is determined by two different points and extends infinitely in **one** direction.		Ray AB or $\overrightarrow{AB}$ Ray DC or $\overrightarrow{DC}$
A line segment is determined by two different points and extends in **no** direction. The two different points are called "endpoints." The distance between the endpoints is called the length of the line segment.		Line segment AB or $\overline{AB}$. The length of $\overline{AB}$ or AB = 5 ft. Note: $\overline{AB}$ refers to the line segment and AB refers to the length.

Solve the following:

Ex 1 Given AC = 16 inches and AB = 9 inches in the diagram below, find BC.

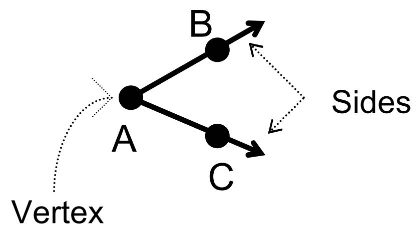


Solution:
Since BC = AC – AB, then BC = 16 – 9 = 7 inches.

Objective b: Understanding types of angles.

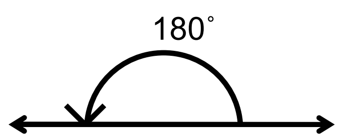
Definition **Illustration** **Notation**

Two rays that share a common endpoint form an angle. Rays AC and AB form angle CAB. The common endpoint, A, is called the vertex of the angle and the two rays are called the sides of the angle. An angle is measured using a tool called a protractor. A protractor is marked in equal increments called degrees. There are 180˚ in an angle whose sides form a straight line.



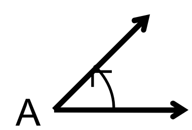
Angle A or ∠ A

Sometimes its useful to use all three points to denote an angle, i.e., “angle BAC” or “∠ CAB.” The vertex is always the middle letter in this notation.



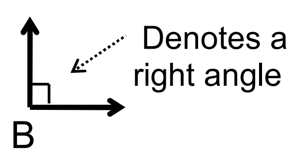
∠ B is a straight angle. The measure of ∠ B, denoted m∠ B, is 180˚. We can say: m∠ B = 180˚

An acute angle is an angle whose measure is between 0˚ and 90˚.



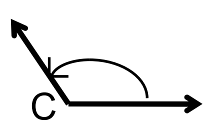
∠ A is an acute angle since m∠ A < 90˚.

A right angle is an angle whose measure is exactly 90˚.



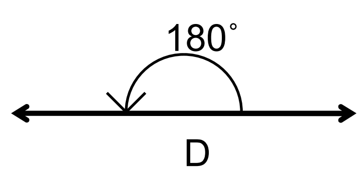
∠ B is a right angle since m∠ B = 90˚.

An obtuse angle is an angle whose measure is between 90˚ and 180˚.



∠ C is an obtuse angle since m∠ C > 90˚ and m∠ C < 180˚.

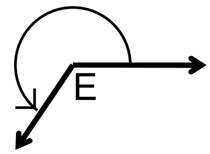
A straight angle is an angle whose measure is exactly 180˚.



∠ D is a straight angle since m∠ D = 180˚.

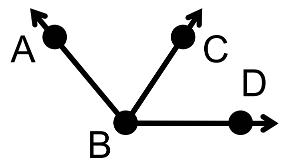
Definition | **Illustration** | **Notation**

A reflex angle is an angle whose measure is between 180˚ and 360˚.



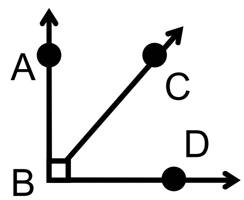
$\angle E$ is a reflex angle since $m\angle E > 180^\circ$ and $m\angle E < 360^\circ$.

Two angles are called adjacent angles if they are side by side and have the same vertex.



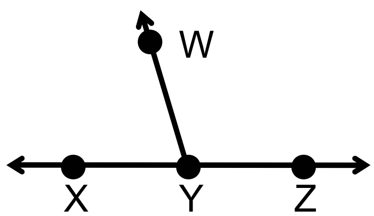
$\angle ABC$ and $\angle CBD$ are adjacent angles since they are side by side and have the point B as their vertex.

Two acute angles are called Complementary angles if the sum of their measures is 90˚. Each is called the "complement" of the other.



$\angle ABC$ and $\angle CBD$ are complementary angles since $m\angle ABC + m\angle CBD = 90^\circ$.

Two angles measuring less than 180˚ are called Supplementary angles if the sum of their measures is 180˚. Each is called the "supplement" of the other.



$\angle XYW$ and $\angle WYZ$ are supplementary angles since $m\angle XYW + m\angle WYZ = 180^\circ$

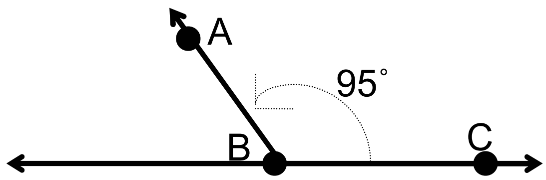
Note that complementary and supplementary angles are not always adjacent angles.

Solve the following:

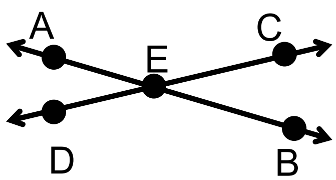
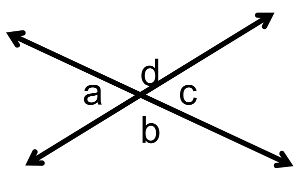
Ex. 2 Given that $m\angle A = 36^\circ$, find the measure of the complement and the supplement of $\angle A$.

Solution:
Since complementary angles total 90˚, then the measure of the complement of $\angle A$ is $90^\circ - m\angle A = 90^\circ - 36^\circ = 54^\circ$.
Since supplementary angles total 180˚, then the measure of the supplement of $\angle A$ is $180^\circ - m\angle A = 180^\circ - 36^\circ = 144^\circ$.

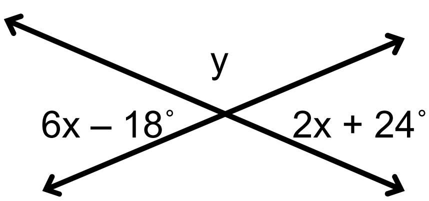
Ex. 3 Given the diagram below, find the measure of the complement and the supplement of ∠ ABC



Solution:
Since complementary angles have to be acute angles (less than 90˚) and the m∠ ABC > 90˚, it is not possible for ∠ ABC to have a complement. Hence, ∠ ABC has no complement.
Since supplementary angles add up to 180˚, then the measure of the supplement of ∠ B is 180˚ – m∠ B = 180˚ – 95˚ = 85˚.

Definition	**Illustration**	**Notation**
Two or more lines are called intersecting if they “cross,” “meet,” or share a common point.		The two lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$, intersect at point E.
Two intersecting lines form vertical angles. These angles come in pairs which have equal measures.		The vertical pairs are: ∠ a & ∠ c; ∠ b & ∠ d. m∠ a = m∠ c m∠ b = m∠ d

Ex. 4 Find the value of x and y in the following diagram:



Solution:
Since 6x – 18˚ and 2x + 24˚ are vertical angles, they are equal.

$$\begin{aligned} 6x - 18^\circ &= 2x + 24^\circ \\ -2x \quad &= -2x \\ \hline 4x - 18^\circ &= 24^\circ \end{aligned}$$

$$4x - 18^\circ = 24^\circ$$
$$\underline{\quad + 18^\circ = + 18^\circ}$$
$$4x = 42^\circ$$

$$\frac{4x}{4} = \frac{42^\circ}{4}$$
$$x = 10.5^\circ$$

Now, substitute 10.5° in for x in $2x + 24^\circ$:

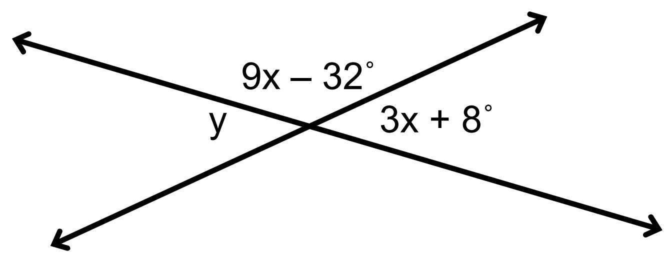
$$2(10.5^\circ) + 24^\circ = 21 + 24 = 45^\circ$$

Since y and $2x + 24^\circ$ are supplementary angles, then

$$y = 180^\circ - 45^\circ = 135^\circ.$$

So, $x = 10.5^\circ$ and $y = 135^\circ$.

Ex. 5 Find the value of x and y in the following diagram:



Solution:

Since $9x - 32^\circ$ and $3x + 8^\circ$ are supplementary angles, their sum is 180°:

$$9x - 32^\circ + 3x + 8^\circ = 180^\circ$$
$$12x - 24^\circ = 180^\circ$$
$$\underline{\quad + 24^\circ = + 24^\circ}$$
$$12x = 204^\circ$$

$$\frac{12x}{12} = \frac{204^\circ}{12}$$

$$x = 17^\circ$$

Since $y = 3x + 8^\circ$ (they are vertical angles), then

$$y = 3x + 8^\circ = 3(17^\circ) + 8^\circ = 51^\circ + 8^\circ = 59^\circ$$

So, $x = 17^\circ$ and $y = 59^\circ$.