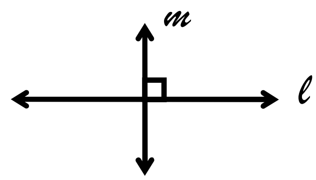
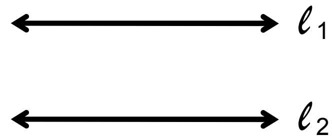
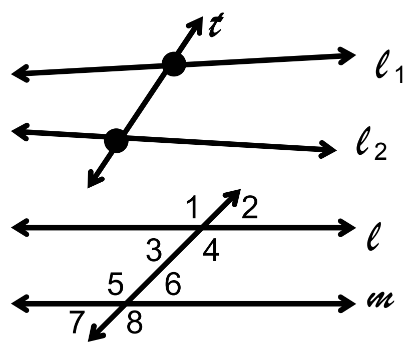
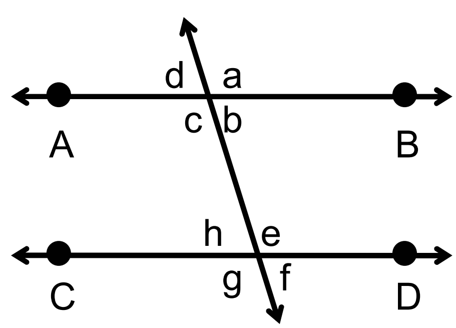
Sect 9.2 - Parallel and Perpendicular Lines

Objective a: Understanding parallel and perpendicular lines.

Definition	**Illustration**	**Notation**
Two intersecting lines (or rays or line segments) that form right angles (90˚) are called <u>perpendicular lines</u>.		Line ℓ is perpendicular to line m or $\ell \perp m$.
Two or more lines are called <u>parallel lines</u> if they never "cross", "meet", or have no points in common.		Lines ℓ_1 and ℓ_2 are parallel or $\ell_1 \parallel \ell_2$.
A <u>transversal</u> is a line that intersects two or more different lines at different points. If the transversal intersects two parallel lines, it produces a total eight angles with some special properties.		Line t is the transversal since it intersects ℓ_1 & ℓ_2 at two different points. If $\ell \parallel m$, then $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$ $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$

Ex. 1 Given that $m\angle b = 63^\circ$ in the diagram below, find all the other angles. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.



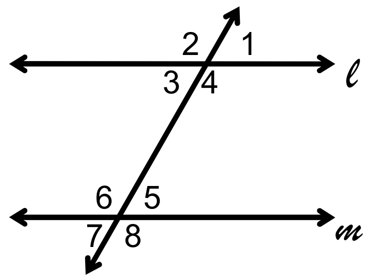
<u>Solution:</u>

Since $m\angle f = m\angle h = m\angle d = m\angle b$, then $m\angle f = m\angle h = m\angle d = 63^\circ$. Since $\angle c$ and $\angle b$ are supplementary angles,
$m\angle c = 180^\circ - m\angle b = 180^\circ - 63^\circ = 117^\circ$.
Hence, $m\angle g = m\angle e = m\angle a = m\angle c = 117^\circ$.

Objective b: Properties of Lines Cut by Transversals.

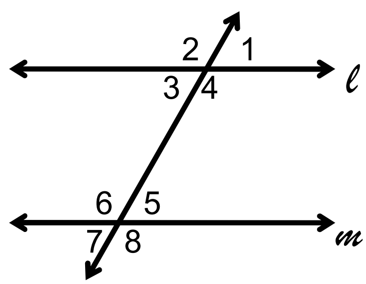
Definition | **Illustration** | **Notation**

Corresponding angles are those angles that share the **same location** in their respective intersections. If two lines that are cut by a transversal are parallel, then the corresponding angles are equal.



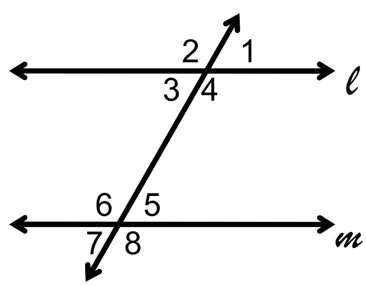
There are four pairs of corresponding angles. They are:
$\angle 1$ & $\angle 5$; $\angle 2$ & $\angle 6$;
$\angle 3$ & $\angle 7$; $\angle 4$ & $\angle 8$.
If $\ell \parallel m$, then
$m\angle 1 = m\angle 5$;
$m\angle 2 = m\angle 6$;
$m\angle 3 = m\angle 7$;
$m\angle 4 = m\angle 8$.

Alternate interior angles are those angles inside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate interior angles are equal.



There are two pairs of alternate interior angles. They are: $\angle 3$ & $\angle 5$;
$\angle 4$ & $\angle 6$.
If $\ell \parallel m$, then
$m\angle 3 = m\angle 5$;
$m\angle 4 = m\angle 6$.

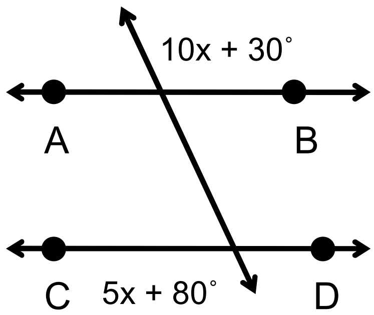
Alternate exterior angles are those angles outside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate exterior angles are equal.



There are two pairs of alternate exterior angles. They are: $\angle 1$ & $\angle 7$;
$\angle 2$ & $\angle 8$.
If $\ell \parallel m$, then
$m\angle 1 = m\angle 7$;
$m\angle 2 = m\angle 8$.

In each of the diagrams, solve for x. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

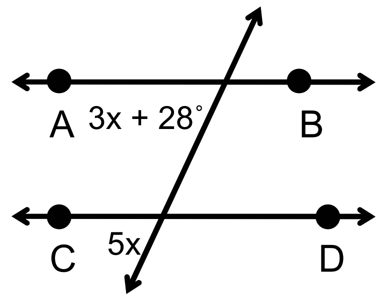
Ex. 2



Solution:
Since the two angles are alternate exterior angles, they are equal. Thus,

$$10x + 30^\circ = 5x + 80^\circ$$
$$-30^\circ = \quad -30^\circ$$
$$10x = 5x + 50^\circ$$
$$-5x = -5x$$
$$\frac{5x}{5} = \frac{50^\circ}{5}$$
$$x = 10^\circ$$

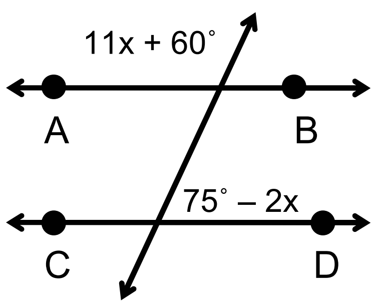
Ex. 3



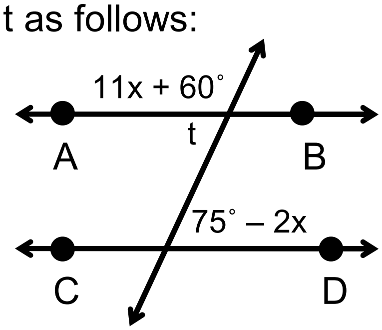
Solution:
Since the two angles are alternate interior angles, they are equal. Thus,

$$5x = 3x + 28^\circ$$
$$-3x \quad -3x$$
$$\frac{2x}{2} = \frac{28^\circ}{2}$$
$$x = 14^\circ$$

Ex. 4

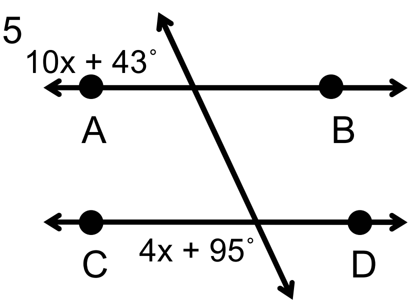


Solution:
Mark t as follows:

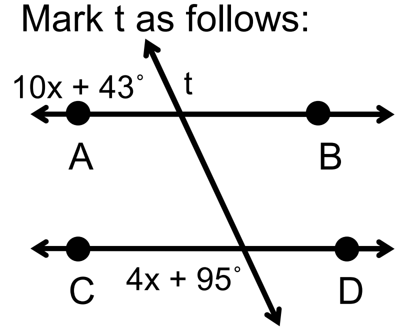


Since $75^\circ - 2x$ and t are alternate interior angles, they are equal and hence,

Ex. 5



Solution:
Mark t as follows:

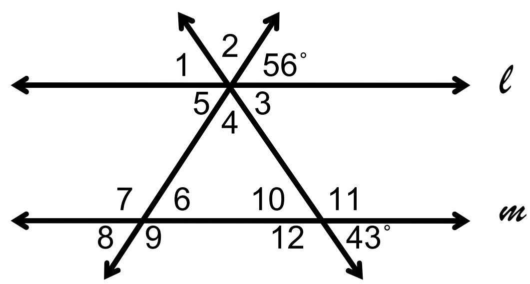


Since $4x + 95^\circ$ and t are alternate exterior angles, they are equal and hence,

we can replace t by $75^\circ - 2x$.
$11x + 60^\circ$ and t are supple - mentary angles, thus
$11x + 60^\circ + t = 180^\circ$
$11x + 60^\circ + 75^\circ - 2x = 180^\circ$
$9x + 135^\circ = 180^\circ$
$\underline{-135^\circ \quad = \quad -135^\circ}$
$\frac{9x}{9} = \frac{45^\circ}{9}$
$x = 5^\circ$

we can replace t by $4x + 95^\circ$.
$10x + 43^\circ$ and t are supple - mentary angles, thus
$10x + 43^\circ + t = 180^\circ$
$10x + 43^\circ + 4x + 95^\circ = 180^\circ$
$14x + 138^\circ = 180^\circ$
$\underline{-138 = \quad -138^\circ}$
$\frac{14x}{14} = \frac{42^\circ}{14}$
$x = 3^\circ$

Ex. 6 In the diagram below, find all the missing angles. Assume $\ell \parallel m$.



Solution:

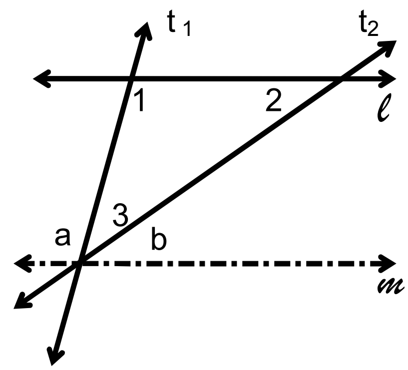
Since $\angle 5$ and 56° are vertical angles, $m\angle 5 = 56^\circ$. Since $\angle 1$ and 43° are alternate exterior angles, $m\angle 1 = 43^\circ$. But $\angle 1$ and $\angle 3$ are vertical angles, thus $m\angle 3 = 43^\circ$. $\angle 1$, $\angle 2$, and 56° form a straight line and $m\angle 1 = 43^\circ$, hence $180^\circ = 56^\circ + m\angle 2 + 43^\circ$. Solving for $m\angle 2$ yields $m\angle 2 = 81^\circ$. Yet, $\angle 2$ and $\angle 4$ are vertical angles which means $m\angle 4 = 81^\circ$. Since $\angle 5$ corresponds to $\angle 8$, $m\angle 8 = m\angle 5 = 56^\circ$. But $\angle 6$ and $\angle 8$ are vertical angles, so $m\angle 6 = 56^\circ$. $\angle 7$ and $\angle 8$ are supplementary angles, therefore $m\angle 7 = 180^\circ - m\angle 8 = 180^\circ - 56^\circ = 124^\circ$. $\angle 7$ and $\angle 9$ are vertical angles and thus $m\angle 9 = 124^\circ$. $\angle 10$ and 43° are vertical angles which means $m\angle 10 = 43^\circ$. Since $\angle 10$ and $\angle 11$ are supplementary angles, $m\angle 11 = 180^\circ - 43^\circ = 137^\circ$. But, $\angle 11$ and $\angle 12$ are vertical angles, so $m\angle 12 = 137^\circ$. Therefore:

$m\angle 1 = 43^\circ$	$m\angle 2 = 81^\circ$	$m\angle 3 = 43^\circ$	$m\angle 4 = 81^\circ$
$m\angle 5 = 56^\circ$	$m\angle 6 = 56^\circ$	$m\angle 7 = 124^\circ$	$m\angle 8 = 56^\circ$
$m\angle 9 = 124^\circ$	$m\angle 10 = 43^\circ$	$m\angle 11 = 137^\circ$	$m\angle 12 = 137^\circ$

In looking back at the diagram in the previous example, observe that the sum of the three angles of the triangle ($m\angle 4 = 81^\circ$, $m\angle 6 = 56^\circ$, and $m\angle 10 = 43^\circ$) is 180°. This is true for any triangle. Thus, the sum of the angles of a triangle is 180°. Let's prove it in general:

Ex. 7 In the triangle below, show that the sum of the measures of the angles is equal to 180˚.

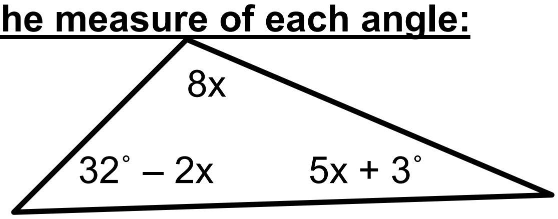
Solution:



Draw m so that it is parallel to ℓ and passes through the vertex of the angle of the triangle not on ℓ. Notice that $\angle a$, $\angle b$, & $\angle 3$ together form a line. This means that $m\angle a + m\angle b + m\angle 3 = 180^\circ$. Since $\ell \parallel m$ and t_1 is a transversal, then $\angle 1$ and $\angle a$ are a pair of alternate interior angles. Hence, $m\angle 1 = m\angle a$ and we can replace $m\angle a$ by $m\angle 1$ in the formula above to get: $m\angle 1 + m\angle b + m\angle 3 = 180^\circ$. But, t_2 is also a transversal, so $\angle 2$ and $\angle b$ are a pair of alternate interior angles. Hence, $m\angle 2 = m\angle b$ and we can replace $m\angle b$ by $m\angle 2$ to get: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.
Thus, the sum of the measures of the angles is 180˚.

<u>Find the measure of each angle:</u>

Ex. 8



Solution:

Since the sum of the angles of a triangle is 180˚, our equation becomes:

$$32^\circ - 2x + 8x + 5x + 3^\circ = 180^\circ$$
$$11x + 35^\circ = 180^\circ$$
$$-35^\circ = -35^\circ$$
$$\frac{11x}{11} = \frac{145^\circ}{11}$$
$$x = 13\tfrac{2}{11}^\circ$$

Now, plug in to find the angles:

$$32 - 2x = 32^\circ - 2\left(\frac{145^\circ}{11}\right) = \frac{32^\circ}{1} - \frac{290^\circ}{11} = \frac{352^\circ}{11} - \frac{290^\circ}{11} = \frac{62^\circ}{11} = 5\tfrac{7}{11}^\circ$$

$$8x = 8\left(\frac{145^\circ}{11}\right) = \frac{1160^\circ}{11} = 105\tfrac{5}{11}^\circ$$

$$5x + 3^\circ = 5\left(\frac{145^\circ}{11}\right) + 3 = \frac{725^\circ}{11} + \frac{3^\circ}{1} = \frac{725^\circ}{11} + \frac{33^\circ}{11} = \frac{758^\circ}{11} = 68\tfrac{10}{11}^\circ$$