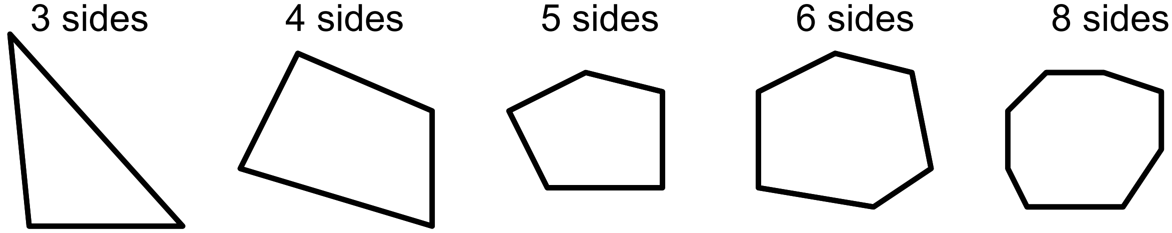
Sect 9.3 - Polygons

Objective a: Understanding and Classifying Different Types of Polygons.

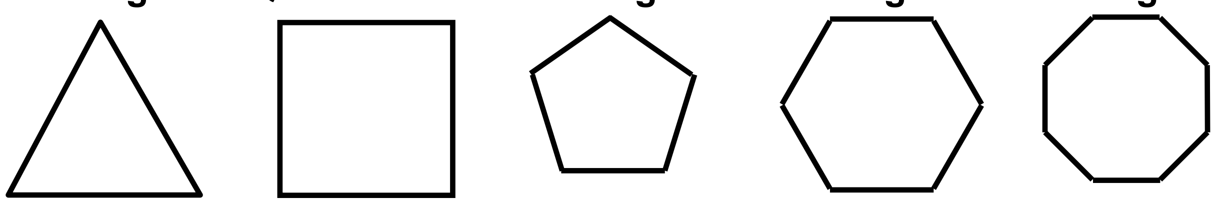
A **Polygon** is closed two-dimensional geometric figure consisting of at least three line segments for its sides. If all the sides are the same length (the angles will also have the same measure), then the figure is called a **Regular Polygon**. The points where the two sides intersect are called vertices.

Triangle	Quadrilateral	Pentagon	Hexagon	Octagon
3 sides	4 sides	5 sides	6 sides	8 sides



Regular Polygons

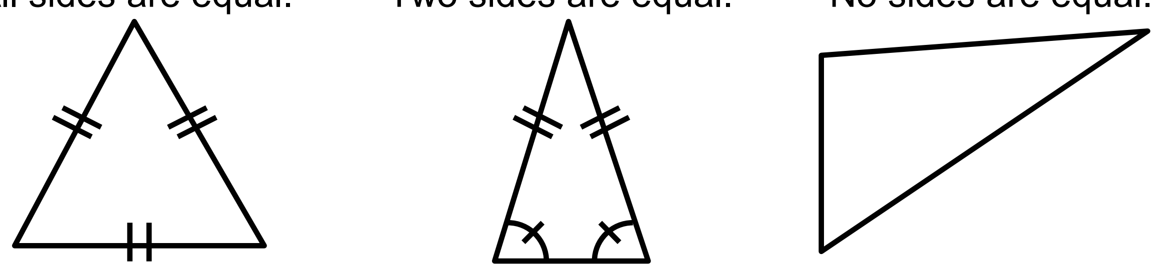
Triangle	Quadrilateral	Pentagon	Hexagon	Octagon



Objective b: Understanding and Classifying Different Types of Triangles.

We can classify triangles by their sides:

Equilateral Triangle	Isosceles Triangle	Scalene Triangle
All sides are equal.	Two sides are equal.	No sides are equal.

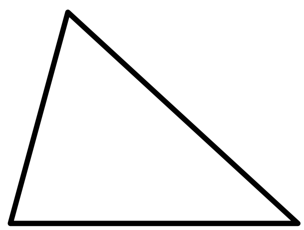


All angles measure 60˚.	The two base angles are equal.	No angles are equal.

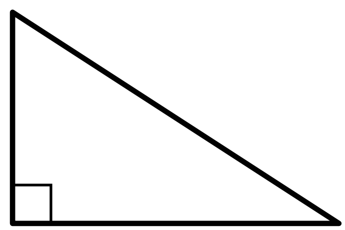
An equilateral triangle is also called an **equiangular triangle**. The third angle in an isosceles triangle formed by the two equal sides is called the **vertex angle**.

We can also classify triangles by their angles:

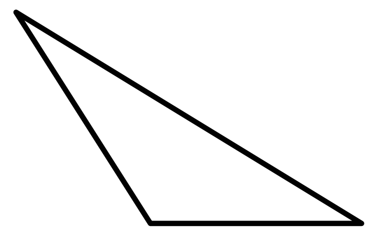
Acute Triangle
All angles are acute.



Right Triangle
Has on right angle.



Obtuse Triangle
Has one obtuse angle.

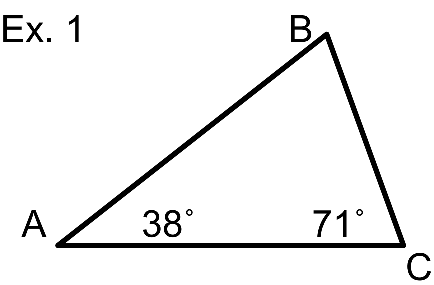


In a right triangle, the two sides that intersect to form the right angle are called the **legs** of the right triangle while the third side (the longest side) of a right triangle is called the **hypotenuse** of the right triangle.

Recall that the sum of the measures of the angles of a triangle is equal to 180˚.

<u>Determine what type of triangle is picture below:</u>

Ex. 1

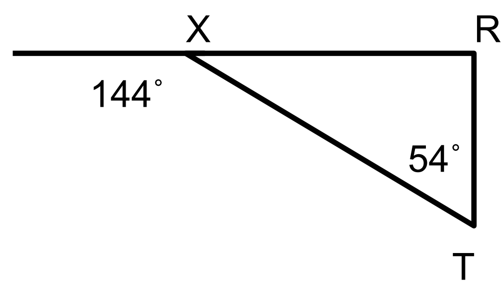


<u>Solution:</u>
Since the measures of the angles of a triangle total 180˚, then

$38^\circ + m\angle B + 71^\circ = 180^\circ$
$m\angle B + 109^\circ = 180^\circ$
$\underline{-109^\circ = -109^\circ}$
$m\angle B = 71^\circ$.

So, ∆ABC is an Isosceles and an Acute Triangle.

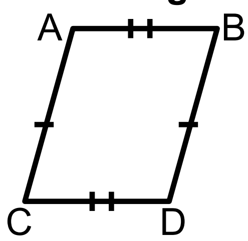
Ex. 2



<u>Solution:</u>
Since ∠TXR and 144˚ are supplementary angles, then $m\angle TXR = 180^\circ - 144^\circ = 36^\circ$. But, $m\angle TXR + m\angle T = 54^\circ + 36^\circ = 90^\circ$. So, $\angle R = 180^\circ - 90^\circ = 90^\circ$. Since the angles are different, then none of the sides are equal. Thus, ∆XRT is a scalene and a right triangle.

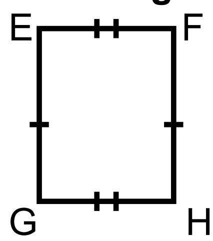
Objective c: Understanding Different Types of Quadrilaterals.

Parallelogram



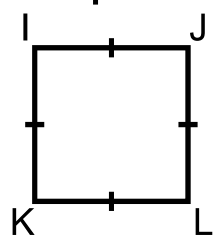
Properties:
AB || CD
AC || BD
AB = CD
AC = BD
m∠A = m∠D
m∠B = m∠C

Rectangle



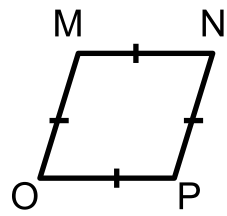
Properties:
EF || GH
EG || FH
EF = GH
EG = FH
All the angles measure 90˚.
The diagonals EH and GF are equal.

Square



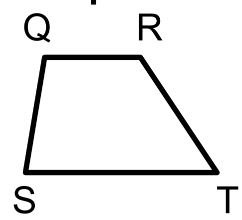
Properties:
IK || JL
IJ || KL
All sides are equal.
All the angles measure 90˚.
The diagonals IL and KJ are equal.

Rhombus



Properties:
MN || OP
MO || NP
All sides are equal.
m∠M = m∠P
m∠N = m∠O

Trapezoid

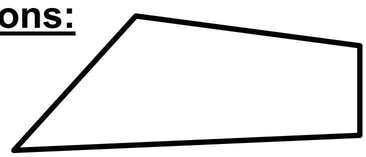


Properties:
Only two sides are parallel:
ST || QR

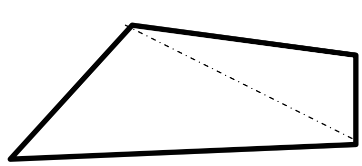
The diagonals of a parallogram are equal if and only if the parallelogram is a rectangle.

<u>Determine the sum of the measures of the angles of the following polygons:</u>

Ex. 3

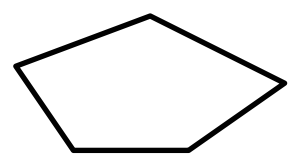


<u>Solution:</u>
We can split any quadrilateral into two triangles:

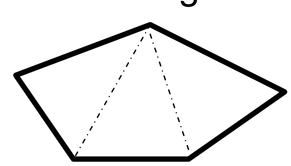


The sum of the angles in each triangle is 180˚, so the sum for quadrilateral is 2•180˚ = 360˚.

Ex. 4



<u>Solution:</u>
We can split any pentagon into three triangles”



The sum of the angles in each triangle is 180˚, so the sum for pentagon is 3•180˚ = 540˚.

A quadrilateral has four sides so it can be split into two triangles and so the sum of the measures of the angles is equal to $2 \bullet 180^\circ$. A pentagon has five sides so it can be split into three triangles and so the sum of the measures of the angles is equal to $3 \bullet 180^\circ$. Notice the pattern; the number of triangles you can divide a polygon is equal to two less than the number of sides of the polygon. So, the sum of the measures of the angles of a polygon is the quantity of the number of sides minus two, times 180°.

Angles of a polygon:
The sum, S, of the measures of the angles of a polygon with n sides is: $S = (n - 2) \bullet 180^\circ$

Determine a) the sum of the measures of the angles of the polygon and b) if it is a regular polygon, find the measure of each angle:

Ex. 5 Octagon

Solution:
a) An octagon has 8 sides,
so $S = (8 - 2) \bullet 180^\circ$
$= 6 \bullet 180^\circ = 1080^\circ$.
So, the sum of the measures of the angles of an octagon is 1080°.
b) Since a regular octagon has eight equal angles, then the measure of each equal angle is: $1080^\circ \div 8 = 135^\circ$.

Ex. 6 Decagon (10 sides)

Solution:
a) An decagon has 10 sides
so $S = (10 - 2) \bullet 180^\circ$
$= 8 \bullet 180^\circ = 1440^\circ$.
So, the sum of the measures of the angles of an decagon is 1440°.
b) Since a regular decagon has ten equal angles, then the measure of each equal angle is: $1440^\circ \div 10 = 144^\circ$.

Given the sum of the measures of the angles of the polygon, find the number of sides the polygon has:

Ex. 7 2160°

Solution:
$S = (n - 2) \bullet 180^\circ$
$2160^\circ = (n - 2) \bullet 180^\circ$
$2160^\circ = 180^\circ n - 360^\circ$
$\underline{+\,360^\circ = \qquad +\,360^\circ}$
$\dfrac{2520^\circ}{180^\circ} = \dfrac{180^\circ n}{180^\circ}$
n = 14 sides.

Ex. 8 900°

Solution:
$S = (n - 2) \bullet 180^\circ$
$900^\circ = (n - 2) \bullet 180^\circ$
$900^\circ = 180^\circ n - 360^\circ$
$\underline{+\,360^\circ = \qquad +\,360^\circ}$
$\dfrac{1260^\circ}{180^\circ} = \dfrac{180^\circ n}{180^\circ}$
n = 7 sides.