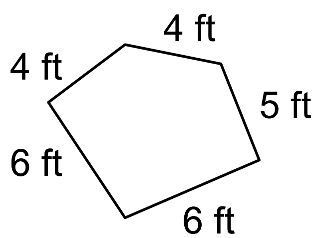
Sect 9.5 - Perimeters and Areas of Polygons

Objective a: Understanding Perimeters of Polygons.

The **Perimeter** is the length around the outside of a closed two - dimensional figure. For a polygon, the perimeter is the sum of the length of the sides of the polygon. We use the idea of perimeter when we calculate how much fencing we need to enclose a garden or the amount of wood we need to frame in a door.

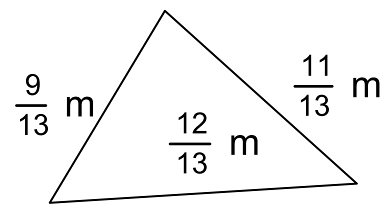
Find the perimeter of the following:

Ex. 1



Solution:
To find the perimeter, simply add up the lengths of the sides:
P = 6 + 4 + 4 + 5 + 6 = 25 ft.

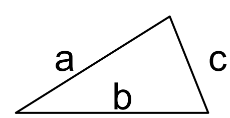
Ex . 2



Solution:
To find the perimeter, simply add up the lengths of the sides:
$P = \frac{9}{13} + \frac{12}{13} + \frac{11}{13} = \frac{32}{13}$ m.

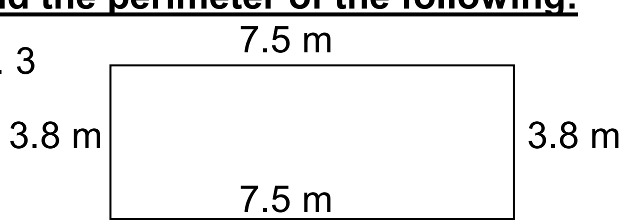
Perimeter of a Triangle

In general, if a, b, and c are the lengths of the sides of a triangle, then the formula for the perimeter of a triangle is $P = a + b + c$.



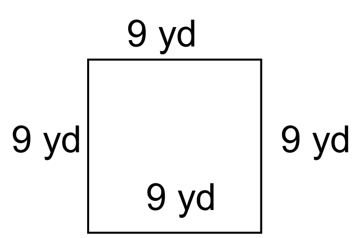
Find the perimeter of the following:

Ex. 3



Solution:
To find the perimeter, simply add up the lengths of the sides:
P = 2(7.5) + 2(3.8) = 22.6 m.

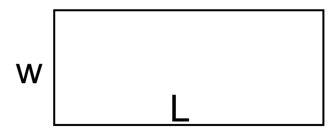
Ex. 4



Solution:
To find the perimeter, simply add up the lengths of the sides:
P = 4(9) = 36 yd.

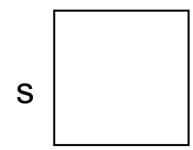
Perimeter of a Rectangle

In general, if L is the length of a rectangle and w is the width of the rectangle, then the formula for the perimeter of a rectangle is P = 2L + 2w.



Perimeter of a Square

In general, if s is the length of the side of a square, the formula for the perimeter of a square is P = 4s.



<u>**Find the following:**</u>

Ex. 5 The perimeter of a rectangle is 52 feet. If the length is 60 inches more than twice the width, find the dimensions of the rectangle.

<u>Solution:</u>

First, convert the 60 inches into the feet:

$$60 \text{ in} = \frac{60\text{in}}{1} \bullet \frac{1\text{ft}}{12\text{in}} = \frac{60}{12} \text{ ft} = 5 \text{ ft.}$$

So, the length is five feet more than twice the width.

Let w = width.

Since the length is five feet more than twice the width, then L = 2w + 5. The perimeter is P = 2L + 2w = 52. Now, replace L by 2w + 5:

$2L + 2w = 52$	
$2(2w + 5) + 2w = 52$	(distribute)
$4w + 10 + 2w = 52$	(combine like terms)
$6w + 10 = 52$	(subtract 10 from both sides)
$-10 = -10$	
$\frac{6w}{6} = \frac{42}{6}$	(divide by 6)

w = 7. Since L = 2w + 5, then L = 2(7) + 5 = 14 + 5 = 19.

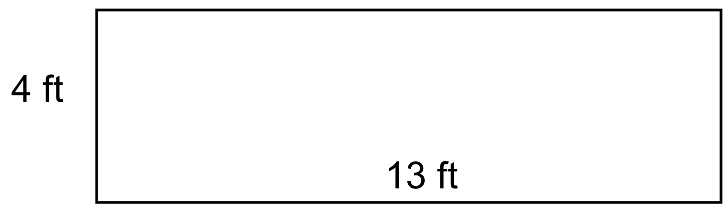
Thus, the dimensions are 19 feet by 7 feet.

Objective b: Understanding Areas of Polygons.

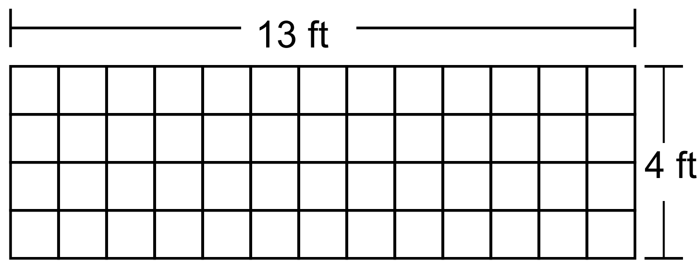
The <u>**Area**</u> is the amount of region inside of closed two – dimensional object. Area is measured in square units such as in^2 or cm^2. We use the idea of area to determine how much paint we need for a wall or how much carpeting we need for a room.

Find the area of the following:

Ex. 6



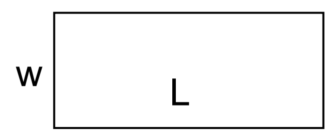
Solution:



To solve this problem, think of how many one foot by one foot tiles you would to tile a floor that is 13 feet by 4 feet. You would need four rows of 13 tiles or $4 \bullet 13 = 52$ tiles. Thus, the area of a the rectangle is 52 ft^2.

Area of a Rectangle

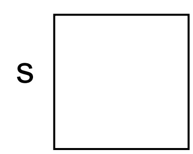
In general, if L is the length of a rectangle and w is the width of the rectangle, then the formula for the area of a rectangle is $A = Lw$.



Since a square is a special rectangle, then its area is $s \bullet s = s^2$.

Area of a Square

In general, if s is the length of the side of a square, the formula for the area of a square is $A = s^2$.



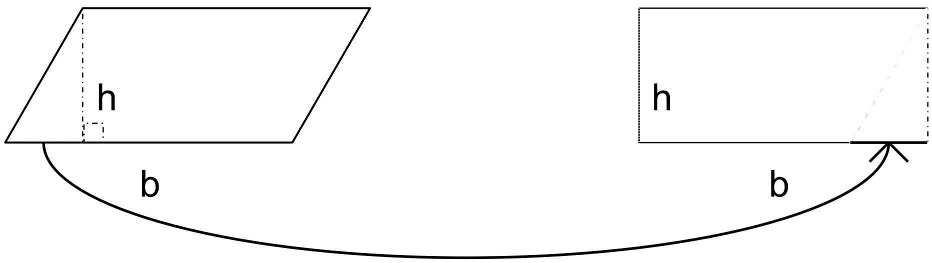
Find the area of the following:

Ex. 7 A square with the length of a side equal to $\frac{2}{3}$ cm.

Solution:

$A = s^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ cm^2.

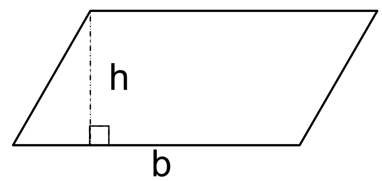
To see where the formula for a parallelogram comes from, we cut the parallelogram along dashed line indicating its height and paste the smaller piece to the other side of the larger piece:



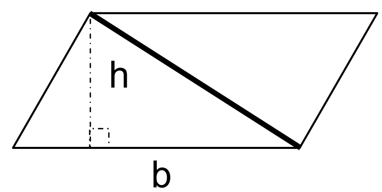
Notice that the resulting figure is a rectangle, but the area of the rectangle is Lw. Since b is equal to L. and h is equal to w, then the area is equal to bh. Thus, the area of a parallelogram is bh.

Area of a Parallelogram

In general, if b is the length of the base of a parallelogram and h is the height of the parallelogram, then the formula for the area of a parallelogram is A = bh.

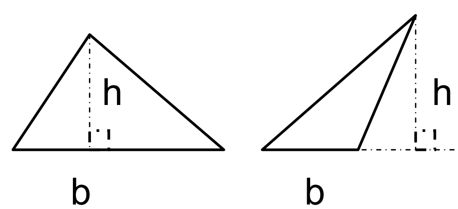


If you divide a parallelogram along one of its diagonals, you get two equal triangles. The areas of both of these equal triangles together is bh, so the area of each is $\frac{1}{2}$bh. This is the formula for the area of a triangle.



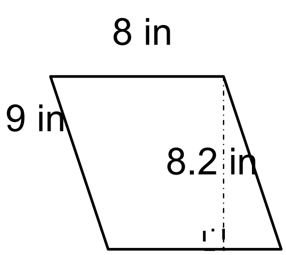
Area of a Triangle

In general, if b is the length of the base of a triangle and h is the height or altitude of the triangle, then the formula for the area of a triangle is A = $\frac{1}{2}$bh.

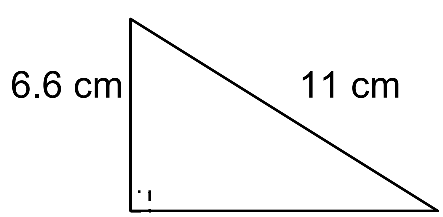


<u>**Find the perimeter and area of the following:**</u>

Ex. 8



Ex. 9

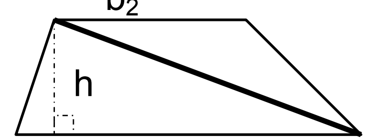


Solution:
To find the perimeter, we need to add the lengths around the outside. We ignore the height of 8.2 in.
P = 9 + 8 + 9 + 8 = 34 in.
To find the area, we use the formula A = bh. The base is 8 in and the height is 8.2 in, so, A = 8(8.2) = 65.6 in^2.

Solution:
We need the Pythagorean Theorem to find the base of the triangle:

$$(11)^2 = (6.6)^2 + b^2$$
$$121 = 43.56 + b^2$$
$$-43.56 = -43.56$$
$$77.44 = b^2$$

So, $b = \sqrt{77.44} = 8.8$ cm.
Thus, P = a + b + c
= 6.6 + 8.8 + 11 = 26.4 cm.
The area is $A = \frac{1}{2}bh$
$= \frac{1}{2}(8.8)(6.6) = 29.04\ cm^2$.



The last figure we need to look at is a trapezoid. We can split a trapezoid along one of its diagonals into two triangles. The height of each triangle is h and the bases are b_1 and b_2 respectively. Thus, the area of a trapezoid is equal to the sum of the areas of these two triangles: $A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$. Notice the formula given for the area of a trapezoid is really the same as this:

$A = \frac{1}{2}(b_1 + b_2)h$ (Commutative property)

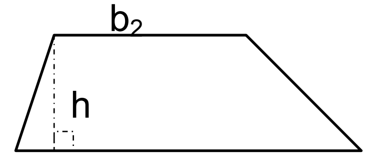
$A = \frac{1}{2}h(b_1 + b_2)$ (Distribute)

$A = \frac{1}{2}hb_1 + \frac{1}{2}hb_2$ (Commutative property)

$A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$.

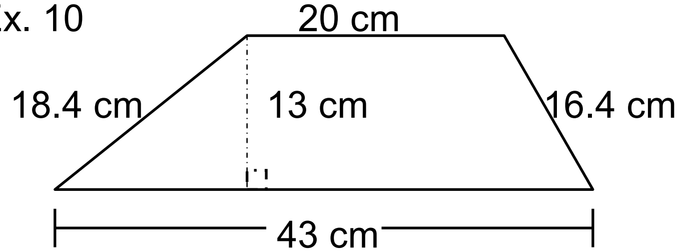
Area of a Trapezoid

In general, if b_1 and b_2 are the lengths of the bases of a trapezoid and h is the height of the trapezoid, then the formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.



Find the perimeter and area of the following:

Ex. 10



Solution:

Perimeter:

Add the lengths around the outside of the figure:

P = 18.4 + 20 + 16.4 + 43

= 97.8 cm.

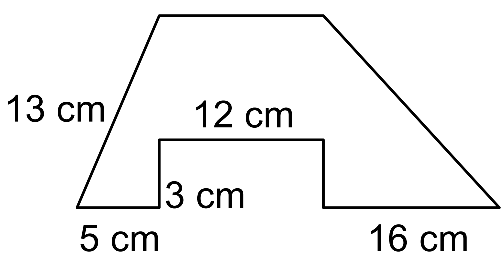
Area:

The bases are 43 and 20 and the height is 13. Plugging in, we get:

$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(43 + 20)13$

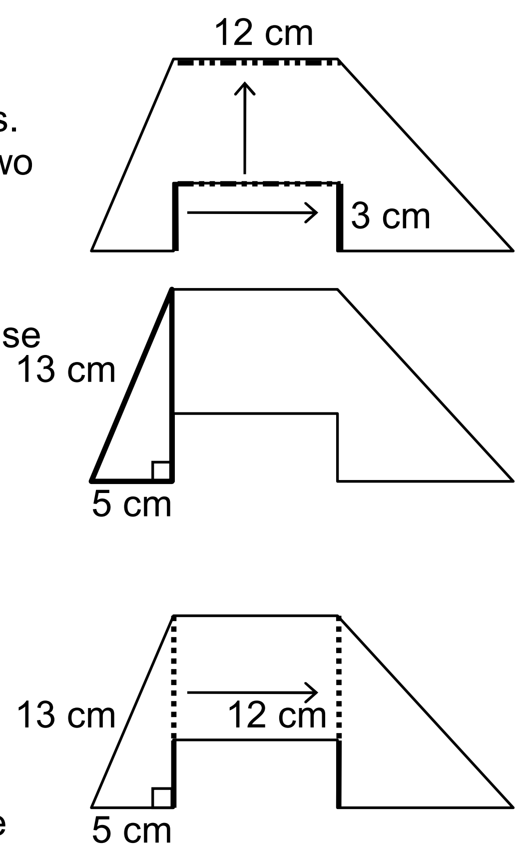
$= 0.5(63)13 = 409.5 \text{ cm}^2$.

Ex. 11



Solution:

We begin by finding the missing sides. Using symmetry, we can easily find two of the missing sides. The other side will prove to be more difficult. Notice that embedded on the left side of the figure is a right triangle with hypotenuse of 13 cm and a leg of 5 cm. We can use the Pythagorean Theorem to find the other leg which will correspond to the height of the figure.



$(13)^2 = (5)^2 + h^2$

$169 = 25 + h^2$

$\underline{-25 = -25}$

$144 = h^2$

$h = \pm\sqrt{144} = \pm 12$

So, the height of the figure is 12 cm. But embedded on the right side of the figure is another right triangle whose base is 16 cm and height is 12 cm. Thus, we have two legs of a triangle and are looking for the hypotenuse:

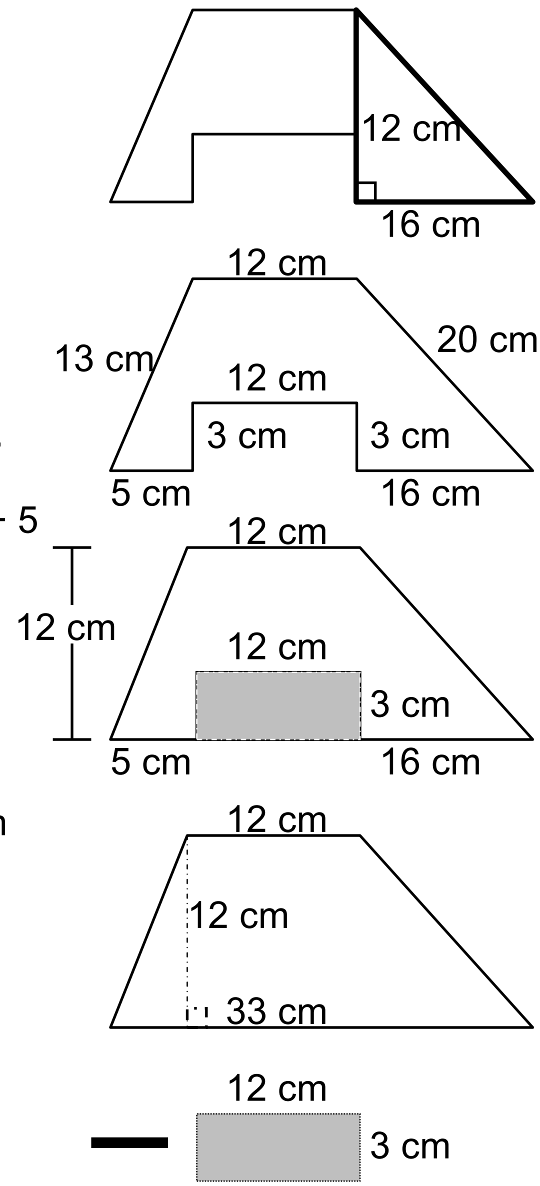
$c^2 = a^2 + b^2$
$c^2 = (12)^2 + (16)^2$
$c^2 = 144 + 256$
$c^2 = 400$
$c = \pm\sqrt{400} = \pm 20$

So, the hypotenuse is 20 cm. Now, we have all the sides of the polygon. Thus, the perimeter is:

$P = 13 + 12 + 20 + 16 + 3 + 12 + 3 + 5$
$= 84$ cm.

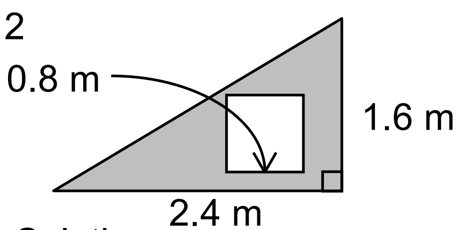
To get the area, think of this as a trapezoid with a rectangle cut out of it. The length of the base along the bottom is 5 + 12 + 16 = 33 cm. The base on the top is 12 cm. The height of the trapezoid is 12 cm from what we found from before. The rectangle is 12 cm by 3 cm. So, we calculate the area of each of these figures and subtract:

$A = \frac{1}{2}(b_1 + b_2)h - Lw$
$= \frac{1}{2}(33 + 12)12 - (12)(3)$
$= 270 - 36 = 234\ \text{cm}^2$



Find the area of the shaded region:

Ex. 12



Solution:

Find the area of the triangle minus the area of the square:

$A = \frac{1}{2}bh - s^2 = \frac{1}{2}(2.4)(1.6) - (0.8)^2$
$= 1.92 - 0.64 = 1.28\ \text{m}^2$.