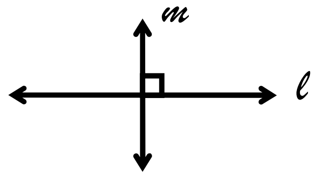
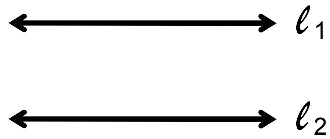
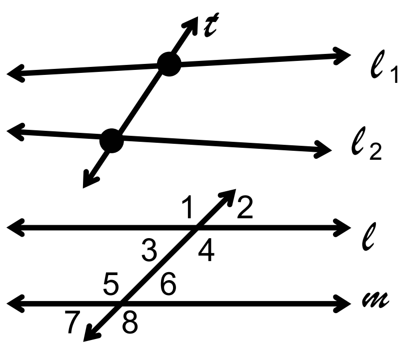
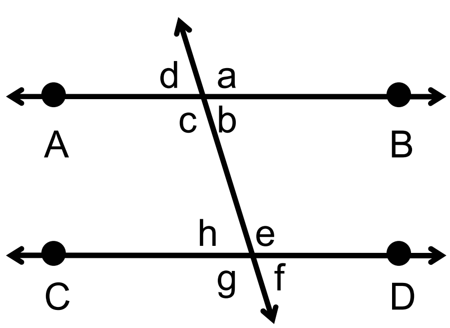
Sect 9.2 - Parallel and Perpendicular Lines

Objective a: Understanding parallel and perpendicular lines.

Definition	**Illustration**	**Notation**
Two intersecting lines (or rays or line segments) that form right angles (90˚) are called perpendicular lines.		Line ℓ is perpendicular to line m or $\ell \perp m$.
Two or more lines are called parallel lines if they never "cross", "meet", or have no points in common.		Lines ℓ_1 and ℓ_2 are parallel or $\ell_1 \parallel \ell_2$.
A transversal is a line that intersects two or more different lines at different points. If the transversal intersects two parallel lines, it produces a total eight angles with some special properties.		Line t is the transversal since it intersects ℓ_1 & ℓ_2 at two different points. If $\ell \parallel m$, then $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$, $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$

Ex. 1 Given that $m\angle b = 63˚$ in the diagram below, find all the other angles. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

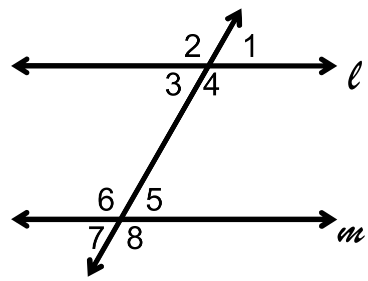


Solution:
Since $m\angle f = m\angle h = m\angle d = m\angle b$, then $m\angle f = m\angle h = m\angle d = 63˚$. Since $\angle c$ and $\angle b$ are supplementary angles,
$m\angle c = 180˚ – m\angle b = 180˚ – 63˚ = 117˚$.
Hence, $m\angle g = m\angle e = m\angle a = m\angle c = 117˚$.

Objective b: Properties of Lines Cut by Transversals.

Definition	**Illustration**	**Notation**

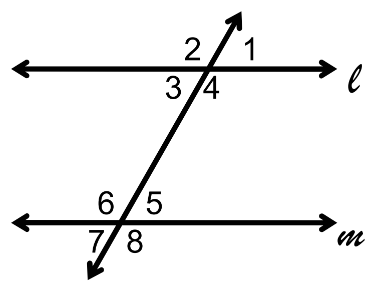
Corresponding angles are those angles that share the **same location** in their respective intersections. If two lines that are cut by a transversal are parallel, then the corresponding angles are equal.



There are four pairs of corresponding angles. They are:
$\angle 1$ & $\angle 5$; $\angle 2$ & $\angle 6$;
$\angle 3$ & $\angle 7$; $\angle 4$ & $\angle 8$.

If $\ell \parallel m$, then
$m\angle 1 = m\angle 5$;
$m\angle 2 = m\angle 6$;
$m\angle 3 = m\angle 7$;
$m\angle 4 = m\angle 8$.

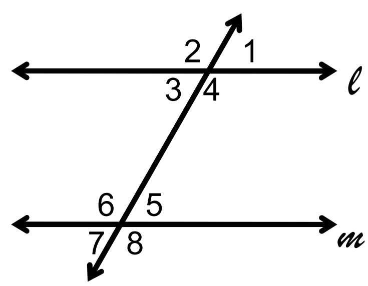
Alternate interior angles are those angles inside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate interior angles are equal.



There are two pairs of alternate interior angles. They are: $\angle 3$ & $\angle 5$;
$\angle 4$ & $\angle 6$.

If $\ell \parallel m$, then
$m\angle 3 = m\angle 5$;
$m\angle 4 = m\angle 6$.

Alternate exterior angles are those angles outside the two lines sharing the opposite locations, i.e., top - left and bottom - right. If the two lines are parallel, then the alternate exterior angles are equal.

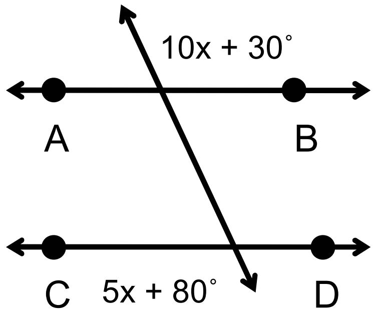


There are two pairs of alternate exterior angles. They are: $\angle 1$ & $\angle 7$;
$\angle 2$ & $\angle 8$.

If $\ell \parallel m$, then
$m\angle 1 = m\angle 7$;
$m\angle 2 = m\angle 8$.

<u>In each of the diagrams, solve for x. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.</u>

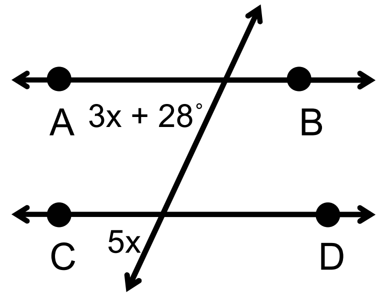
Ex. 2



<u>Solution:</u>
Since the two angles are alternate exterior angles, they are equal. Thus,

$$10x + 30^\circ = 5x + 80^\circ$$
$$\underline{\quad -30^\circ = \quad -30^\circ}$$
$$10x = 5x + 50^\circ$$
$$\underline{-5x = -5x}$$
$$\frac{5x}{5} = \frac{50^\circ}{5}$$
$$x = 10^\circ$$

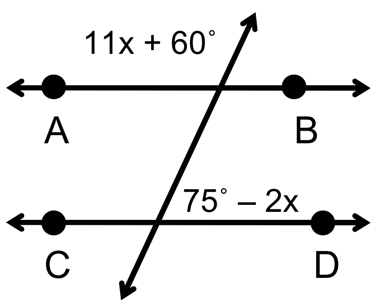
Ex. 3



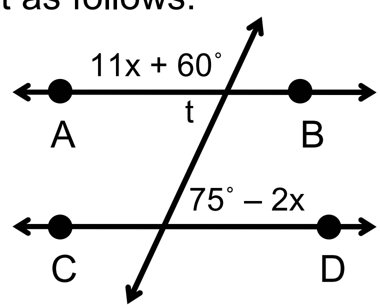
<u>Solution:</u>
Since the two angles are alternate interior angles, they are equal. Thus,

$$5x = 3x + 28^\circ$$
$$\underline{-3x \quad -3x}$$
$$\frac{2x}{2} = \frac{28^\circ}{2}$$
$$x = 14^\circ$$

Ex. 4

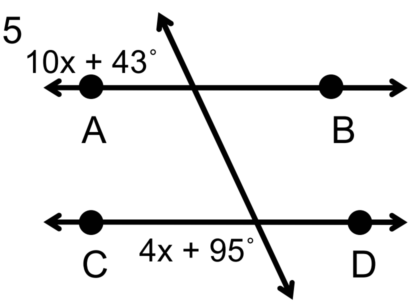


<u>Solution:</u>
Mark t as follows:

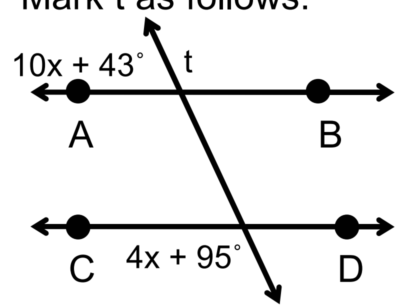


Since 75˚ – 2x and t are alternate interior angles, they are equal and hence,

Ex. 5



<u>Solution:</u>
Mark t as follows:



Since 4x + 95˚ and t are alternate exterior angles, they are equal and hence,

we can replace t by $75^\circ - 2x$.
$11x + 60^\circ$ and t are supple - mentary angles, thus

$11x + 60^\circ + t = 180^\circ$

$11x + 60^\circ + 75^\circ - 2x = 180^\circ$

$9x + 135^\circ = 180^\circ$

$\underline{-135^\circ \quad = \quad -135^\circ}$

$\frac{9x}{9} = \frac{45^\circ}{9}$

$x = 5^\circ$

we can replace t by $4x + 95^\circ$.
$10x + 43^\circ$ and t are supple - mentary angles, thus

$10x + 43^\circ + t = 180^\circ$

$10x + 43^\circ + 4x + 95^\circ = 180^\circ$

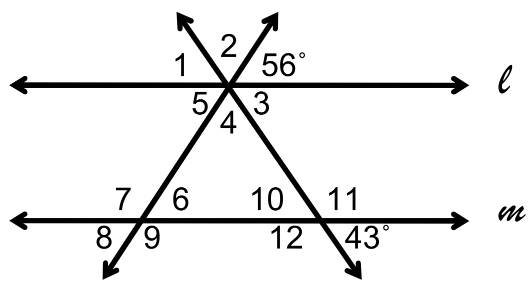
$14x + 138^\circ = 180^\circ$

$\underline{-138 = \quad -138^\circ}$

$\frac{14x}{14} = \frac{42^\circ}{14}$

$x = 3^\circ$

Ex. 6 In the diagram below, find all the missing angles. Assume $\ell \parallel m$.



Solution:

Since ∠ 5 and 56˚ are vertical angles, m∠ 5 = 56˚. Since ∠ 1 and 43˚ are alternate exterior angles, m∠ 1 = 43˚. But ∠ 1 and ∠ 3 are vertical angles, thus m∠ 3 = 43˚. ∠ 1, ∠ 2, and 56˚ form a straight line and m∠ 1 = 43˚, hence 180˚ = 56˚ + m∠ 2 + 43˚. Solving for m∠ 2 yields m∠ 2 = 81˚. Yet, ∠ 2 and ∠ 4 are vertical angles which means m∠ 4 = 81˚. Since ∠ 5 corresponds to ∠ 8, m∠ 8 = m∠ 5 = 56˚. But ∠ 6 and ∠ 8 are vertical angles, so m∠ 6 = 56˚. ∠ 7 and ∠ 8 are supplementary angles, therefore m∠ 7 = 180˚ – m∠ 8 = 180˚ – 56˚ = 124˚. ∠ 7 and ∠ 9 are vertical angles and thus m∠ 9 = 124˚. ∠ 10 and 43˚ are vertical angles which means m∠ 10 = 43˚. Since ∠ 10 and ∠ 11 are supplementary angles, m∠ 11 = 180˚ – 43˚ = 137˚. But, ∠ 11 and ∠ 12 are vertical angles, so m∠ 12 = 137˚. Therefore:

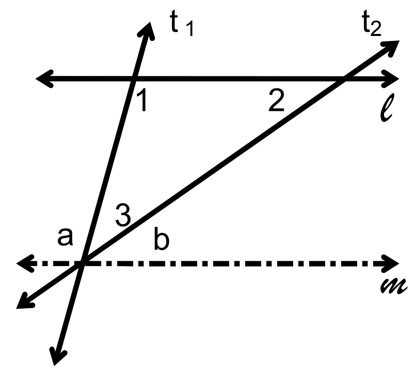
m∠ 1 = 43˚	m∠ 2 = 81˚	m∠ 3 = 43˚	m∠ 4 = 81˚
m∠ 5 = 56˚	m∠ 6 = 56˚	m∠ 7 = 124˚	m∠ 8 = 56˚
m∠ 9 = 124˚	m∠ 10 = 43˚	m∠ 11 = 137˚	m∠ 12 = 137˚

In looking back at the diagram in the previous example, observe that the sum of the three angles of the triangle (m∠ 4 = 81˚, m∠ 6 = 56˚, and m∠ 10 = 43˚) is 180˚. This is true for any triangle. Thus, the sum of the angles of a triangle is 180˚. Let’s prove it in general:

Ex. 7 In the triangle below, show that the sum of the measures of the angles is equal to 180˚.

Solution:

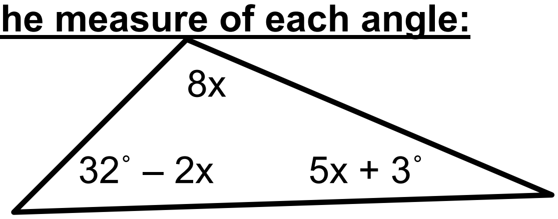
Draw m so that it is parallel to ℓ and passes through the vertex of the angle of the triangle not on ℓ. Notice that ∠ a, ∠ b, & ∠ 3 together form a line. This means that m∠ a + m∠ b + m∠ 3 = 180˚. Since $\ell \parallel m$ and t_1 is a transversal, then ∠ 1 and ∠ a are a pair of alternate interior angles. Hence, m∠ 1 = m∠ a and we can replace m∠ a by m∠ 1 in the formula above to get:



m∠ 1 + m∠ b + m∠ 3 = 180˚. But, t_2 is also a transversal, so ∠ 2 and ∠ b are a pair of alternate interior angles. Hence, m∠ 2 = m∠ b and we can replace m∠ b by m∠ 2 to get:
m∠ 1 + m∠ 2 + m∠ 3 = 180˚.
Thus, the sum of the measures of the angles is 180˚.

Find the measure of each angle:

Ex. 8



Solution:

Since the sum of the angles of a triangle is 180˚, our equation becomes:

$$32^\circ - 2x + 8x + 5x + 3^\circ = 180^\circ$$
$$11x + 35^\circ = 180^\circ$$
$$-35^\circ = -35^\circ$$
$$\frac{11x}{11} = \frac{145^\circ}{11}$$
$$x = 13\frac{2}{11}^\circ$$

Now, plug in to find the angles:

$$32 - 2x = 32^\circ - 2\left(\frac{145^\circ}{11}\right) = \frac{32^\circ}{1} - \frac{290^\circ}{11} = \frac{352^\circ}{11} - \frac{290^\circ}{11} = \frac{62^\circ}{11} = 5\frac{7}{11}^\circ$$
$$8x = 8\left(\frac{145^\circ}{11}\right) = \frac{1160^\circ}{11} = 105\frac{5}{11}^\circ$$
$$5x + 3^\circ = 5\left(\frac{145^\circ}{11}\right) + 3 = \frac{725^\circ}{11} + \frac{3^\circ}{1} = \frac{725^\circ}{11} + \frac{33^\circ}{11} = \frac{758^\circ}{11} = 68\frac{10}{11}^\circ$$