Appendix #1 - Mixed Units of Measurement

Objective a: Converting between units and mixed units.

Many times, we see measurements expressed with mixed units. When a baby is born, the weight may be expressed as 8 pounds and 11 ounces or more simply as 8 lb 11 oz instead of 139 oz. Converting between units and mixed units is similar to converting between improper fractions and mixed numbers. Let us try some examples:

Convert each unit as indicated:

Ex. 1   1205 in to _____ yd _____ ft _____ in.
   Solution:
   Since 1 ft = 12 in, we will multiply $\frac{1205\text{in}}{1}$ by $\frac{1\text{ft}}{12\text{in}}$, but instead of

   simplifying, we will treat this as a long division problem:

   $$\frac{1205\text{in}}{1} \bullet \frac{1\text{ft}}{12\text{in}} = \frac{1205}{12} \text{ ft}$$

   $$\begin{array}{r} 100 \\ 12\overline{\smash{)}\,1205} \\ -\underline{1200} \\ 5 \end{array}$$

   So, our answer is 100 R 5.
   The 100 corresponds to the number of feet and the remainder is how many inches are leftover.

   Thus, 1205 in = __100_ ft _5_ in.
   Now, convert 100 ft to _____ yd _____ ft.
   Since 3 ft = 1 yd, we will multiply $\frac{100\text{ft}}{1}$ by $\frac{1\text{yd}}{3\text{ft}}$, but instead of

   simplifying, we will treat this as a long division problem:

   $$\frac{100\text{ft}}{1} \bullet \frac{1\text{yd}}{3\text{ft}} = \frac{100}{3} \text{ yd}$$

   $$\begin{array}{r} 33 \\ 3\overline{\smash{)}\,100} \\ -\underline{99} \\ 1 \end{array}$$

   So, our answer is 33 R 1.
   The 33 corresponds to the number of yards and the remainder is how many feet are leftover.

   Thus, 100 ft = _33_ yd _1_ ft and hence,
   1205 in = __100_ ft _5_ in = _33_ yd _1_ ft _5_ in.

Ex. 2   Convert 2 gal 3 qt 1 c to ___ fl oz.
   Solution:
   Now, we are going in the opposite direction. We will convert 2 gal to qt and add 3 qt to our result:

$$2 \text{ gal} = \frac{2\,gal}{1} = \frac{2\,gal}{1} \cdot \frac{4\,qt}{1\,gal} = 8 \text{ qt.}$$

So, 2 gal 3 qt 1 c = 8 qt 3 qt 1 c = 11 qt 1 c.
Next, we will convert the 11 qt to c and add 1 c to our result:

$$11 \text{ qt} = \frac{11\,qt}{1} = \frac{11\,qt}{1} \cdot \frac{4\,c}{1\,qt} = 44 \text{ c.}$$

So, 2 gal 3 qt 1 c = 8 qt 3 qt 1 c = 11 qt 1 c = 44 c + 1 c = 45 c.
Finally, we will convert the 45 c to fl oz:

$$45 \text{ c} = \frac{45\,c}{1} \cdot \frac{8\,floz}{1c} = 360 \text{ fl oz.}$$

Objective b: Operations with mixed units.

With adding and subtracting mixed units, the key is to align like units and treat them almost like combining like terms. The only difference is we may have a carry if we are adding (i.e., 7 in + 9 in = 16 in = 1 ft 4 in, the 1 ft would carry over to the feet column) and we may have to borrow if we are subtracting (i.e., 7 ft 5 in – 9 in = 6 ft + **1 ft** 5 in – 9 in
= 6 ft + **12 in** + 5 in – 9 in = 6 ft 17 in – 9 in = 6 ft 8 in).

Perform the indicated operation:

Ex. 3
```
   3 qt  2 c  7 fl oz
+  5 qt  3 c  5 fl oz
```

Solution:
First add and then take care of the carries:

```
   3 qt  2 c   7 fl oz        But, 12 fl oz = 1 c 4 fl oz, so
+  5 qt  3 c   5 fl oz        add one to the 5 c:
   8 qt  5c   12 fl oz        But, 6 c = 1 qt 2 c, so
=  8 qt  6c    4 fl oz        add one to the 8 qt:
=  9 qt  2 c   4 fl oz.
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Ex. 4
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   8 yd  1 ft   5 in
–  4 yd  2 ft   9 in
```

Solution:
Borrow one from the 1 ft, convert it to 12 in and add to the 5 in:

```
                0 ft + 12 in

   8 yd  1̶ ̶f̶t̶  5 in          →        8 yd  0 ft   17 in
–  4 yd  2 ft   9 in                 –  4 yd  2 ft    9 in
                                                      8 in
```

Now, borrow one from the 8 yd, convert it to 3 ft and add it to the 0 ft. Then finish the problem:

7 yd + *3 ft*

~~8 yd~~ 0 ft   17 in   →   7 yd 3 ft 17 in
 − 4 yd 2 ft   9 in   − 4 yd 2 ft 9 in
              8 in   3 yd 1 ft 8 in

Ex. 5   (1 ton 850 lb 15 oz) × 6

Solution:

First, distribute: (1 ton 850 lb 15 oz) × **6**

= 1 Ton × **6** 850 lb × **6** 15 oz × **6** = 6 ton 5100 lb 90 oz

Now, convert 90 oz into ___ lb ___ oz:

$\dfrac{90\,oz}{1} \cdot \dfrac{1\,lb}{16\,oz} = \dfrac{90}{16}$ lb, and then divide:

$$16 \overline{\smash{\big)}\,90} \quad\begin{array}{r}5\\-80\\\hline 10\end{array}$$

So, 90 oz = __5__ lb __10__ oz. Add
5 lb to the 5100 lb to get:

6 ton 5100 lb 90 oz = 6 ton 5105 lb 10 oz.

Now, convert 5105 lb to ___ ton ___ lb:

$\dfrac{5105\,lb}{1} \cdot \dfrac{1\,ton}{2000\,lb} = \dfrac{5105}{2000}$, and then divide:

$$2000 \overline{\smash{\big)}\,5105} \quad\begin{array}{r}2\\-4000\\\hline 1105\end{array}$$

So, 5105 lb = 2 ton 1105 lb. Add
2 ton to the 6 ton to get:

6 ton 5100 lb 90 oz = 6 ton 5105 lb 10 oz = 8 ton 1105 lb 10 oz.

Ex. 6   (27 days 6 hr 8 min) ÷ 11

Solution:

We begin by dividing 11 into 27 days:

$$\begin{array}{r}2\ \text{days}\\11\overline{\smash{\big)}\,27\ \text{days}\ \ 6\ \text{hr}\ \ 8\ \text{min}}\\-22\ \text{days}\\\hline 5\ \text{days}\ \ 6\ \text{hr}\end{array}$$

But, 5 days = 5(24) = 120 hr,
so we need to add 120 hr to 6 hr
and continue the division:
5 days + 6 hr = 120 hr + 6 hr = 126 hr.

Now, divide 126 hr by 11:

$$\begin{array}{r}2\ \text{days}\ 11\ \text{hr}\\11\overline{\smash{\big)}\,27\ \text{days}\ \ 6\ \text{hr}\ \ 8\ \text{min}}\\-22\ \text{days}\\\hline 5\ \text{days}\ \ 6\ \text{hr}\\= 126\ \text{hr}\\-121\ \text{hr}\\\hline 5\ \text{hr}\ \ 8\ \text{min}\end{array}$$

But, 5 hr = 5(60) = 300 min,
so we need to add 300 min to 8 min
and continue the division:
5 hr + 8 min = 300 min + 8 min
= 308 min.

Now, divide 308 min by 11:

$$\begin{array}{r} \underline{2 \text{ days } 11 \text{ hr } 28 \text{ min}} \\ 11 | \ 27 \text{ days } 6 \text{ hr } 8 \text{ min} \end{array}$$

$$\begin{array}{r} \underline{-\ 22 \text{ days}} \\ 5 \text{ days } 6 \text{ hr} \\ = 126 \text{ hr} \\ \underline{-\ 121 \text{ hr}} \\ 5 \text{ hr } 8 \text{ min} \\ = 308 \text{ min} \\ \underline{-\ 308 \text{ min}} \\ 0 \end{array}$$

Ex. 7   75.3 hm – 546 m

<u>Solution:</u>

Here, we need to either convert the hm to m or the m to hm. If we convert the hm to m, we get: 75.3 hm = 75.30 = 7530 m

So, 75.3 hm – 546 m = 7530 – 546 = 6984 m

Ex. 8   (9 hl 73 L 81 cl) × **5**

<u>Solution:</u>

First distribute: (9 hl 73 L 81 cl) × **5** = 9 hl × **5** + 73 L × **5** + 81 cl × **5**
= 45 hl 365 L 405 cl. Now, convert 405 cl to ___ L ___ cl:
405 cl = 400 cl + 5 cl = 400 + 5 cl = 4 L + 5 cl.

Now, add the 4 L to 365 L. So, 45 hl 365 L 405 cl = 45 hl 369 L 5 cl.
Next, convert 369 L to ___ hl ___ L:
369 L = 300 L + 69 L = 300 + 69 L = 3 hl + 69 L.

Now, add the 3 hl to 45 hl. So, 45 hl 369 L 5 cl = 48 hl 69 L 5 cl.

Ex. 9   Convert $\dfrac{3\,dm}{7\,cg}$ to $\dfrac{m}{g}$.

<u>Solution</u>

First, convert the cg into g:

k   h   da   grams   d   c   m
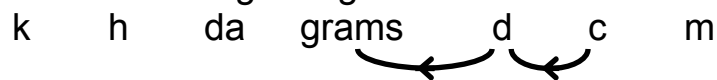
7 cg = 007 = 0.07 g

Next, convert dm to m:

k   h   da   meters   d   c   m

3 dm = 03   = 0.3 dm

Thus, $\frac{3\,dm}{7\,cg} = \frac{0.3\,m}{0.07\,g}$. But, we need to get rid of the decimals so move the decimal point over two places to the right:

$\frac{0.3\,m}{0.07\,g} = \frac{30\,m}{7\,g}$.

Ex. 10   Convert $\frac{1.33\,euro}{qt}$ to $\frac{\$}{gal}$ (hint: 1 euro = $1.20)

Solution:
First, convert the euro into $:

$\frac{1.33\,euro}{qt} = \frac{1.33\,euro}{qt} \bullet \frac{\$1.20}{1\,euro} = \frac{\$1.596}{qt}$.

Now, convert the qt to gal:

$\frac{\$1.596}{qt} = \frac{\$1.596}{qt} \bullet \frac{4\,qt}{1\,gal} = \frac{\$6.384}{gal} \approx \frac{\$6.38}{gal}$.

Ex. 11   A crate containing 23 small appliances weighs 150 lb. If an empty crate weighs 14 lb 14 oz, what is the average weight of each appliance?

Solution:
First, we subtract 14 lb 14 oz from the 150 lb:

      149 lb + *16 oz*
      ~~150 lb~~   149 lb 16 oz
  –   14 lb 14 oz   –   14 lb 14 oz
                         135 lb   2 oz

Now, divide by 23:

       5 lb 14 oz   135 ÷ 23 = 5 R 20.
23 | 135 lb   2 oz
  – 115 lb   But 20 lb = $\frac{20\,lb}{1} \bullet \frac{16\,oz}{1\,lb}$ = 320 oz,
    20 lb   2 oz   So, 20 lb 2 oz = 320 oz + 2 oz = 322 oz
     = 322 oz   322 ÷ 23 = 14 R 0
     – 322 oz
        0

So the average weight of each appliance is 5 lb 14 oz.