**Factoring Trinomials Using Trial and Error**

To factor trinomials where the coefficient of the square term is not 1, we will use have to look at the factors of both a and c and use a technique called Trial & Error. Here is the procedure:

**Factoring** \(ax^2 + bx + c\)

1) List all pairs of factors \(d_1\) and \(d_2\) of \(a\) and all pairs of factors \(e_1\) and \(e_2\) of \(c\).

2) Form all possible binomials of the form \((d_1x + e_1)(d_2x + e_2)\) or \((d_1x + e_1y)(d_2x + e_2y)\) if the original trinomial was in the form \(ax^2 + bxy + cy^2\).

3) Find the sum of the Outer and Inner part of FOIL for each combination found in step #2 until you find a combination that gives you the correct middle term. If you get the opposite sign for the middle term, change the signs of \(e_1\) and \(e_2\). There is a higher probability that the correct combination will involve the factors that are closest to each other in value.

4) If no such product can be found in step #3, the trinomial is prime.

**Examples:**

**Factor the following completely:**

a) \(6x^2 + 11x - 35\)

The possible combinations are

- \((x - 1)(6x + 35), (x + 35)(6x - 1), (x - 5)(6x + 7), (x + 7)(6x - 5), (2x - 1)(3x + 35), (2x + 35)(3x - 1), (2x - 5)(3x + 7), \& (2x + 7)(3x - 5)\)

\((2x - 5)(3x + 7)\) \n\((2x + 7)(3x - 5)\)

\(O. \quad 14x\) \n\(l. \quad 15x\)

\(-x \quad No\) \n\(l. \quad 21x\)

Therefore, \(6x^2 + 11x - 35 = (2x + 7)(3x - 5)\)

b) \(20xy^2 - 130xyz + 60xz^2\)

G.C.F. = 10x, so \(20xy^2 - 130xyz + 60xz^2 = 10x(2y^2 - 13yz + 6z^2)\)

The possible combinations are

- \((y + z)(2y + 6z), (y + 6z)(2y + z), (y + 2z)(2y + 3z) \& (y + 3z)(2y + 2z)\)
\[(y + 2z)(2y + 3z)\]
\[(y + 3z)(2y + z)\]

O. \(3yz\)  No, since the G.C.F. of \(2y + 2z\) \(\neq 1\).

I. \(4yz\)

7yz  No

\[(y + z)(2y + 6z)\]
\[(y + 6z)(2y + z)\]

No, since the G.C.F. of \(2y + 6z\) \(\neq 1\).

O. \(yz\)

I. \(12yz\)

So, we need to change the signs. \(13yz\)  Yes, wrong sign.

Thus, \(10x(2y^2 - 13yz + 6z^2) = 10x(y - 6z)(2y - z)\)

\[c) \quad 14x^2 + 41x + 15\]

\[\begin{array}{c|c}
14x^2 + 41x + 15 & \\
\hline
x \cdot 14x & 1 \cdot 15 \\
2x \cdot 7x & 3 \cdot 5 \\
\end{array}\]

\[(2x + 3)(7x + 5)\]
\[(2x + 5)(7x + 3)\]

O. \(10x\)  O. \(6x\)

I. \(21x\)  I. \(35x\)

31x  No  41x  Yes

So, \(14x^2 + 41x + 15 = (2x + 5)(7x + 3)\).

\[d) \quad -2xy^2 - 17xy + 24x\]

Since G.C.F. = \(-x\), then \(-2xy^2 - 17xy + 24x = -x(2y^2 + 17y - 24)\)

\[\begin{array}{c|c}
2y^2 + 17y - 24 & \\
\hline
y \cdot 2y & -1 \cdot 24 \\
-2 \cdot 12 & (y - 3)(2y + 8) \\
-3 \cdot 8 & (y + 8)(2y - 3) \\
-4 \cdot 6 & \\
\end{array}\]

O. \(-3y\)  O. \(-y\)

I. \(16y\)  I. \(48y\)

\((y - 2)(2y + 12)\)  \((y + 12)(2y - 2)\)

No, G.C.F. \(\neq 1\).  No, G.C.F. \(\neq 1\).

\((y - 1)(2y + 24)\)  \((y + 24)(2y - 1)\)

No, G.C.F. \(\neq 1\).  O. \(-y\)

I. \(48y\)

47y  No

Thus, \(2y^2 + 17y - 24\) is prime. So, our answer is \(-x(2y^2 + 17y - 24)\).
Problems:

**Factor the following completely:**

1) \(-45x^2 - 3xy + 18y^2\)  
2) \(12m^2 - 11mn - 5n^2\)

3) \(6x^2 + 17x + 10\)  
4) \(18x^2 - 12xv + 8v^2\)

5) \(100p^4 + 210p^2 + 90\)  
6) \(-60r^2 - 160r + 175\)